Relativistic Equations and the Bohr Atomic Model

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Abstract: The respective predictions of the energy levels and eigen states through a single unifying relativistic formula.

Keywords: Bohr’s model, Shroedinger’s formula, special relativity, wave-particle duality.

Figure 1 below shows a resonant isolated system that represents my experimental set-up in a simplified fashion to set out the difference in the value of the frequency of an incoming wave after its way through a medium that can be, for instance, an antenna.

Figure 1.

\[
\text{Incoming frequency } F_I \quad \text{medium} \quad \text{potential well /secondary wave } F_\beta
\]

That system obeys the law of energy and momentum conservation as an isolated system that restores its initial energy balance by releasing a secondary wave such as \( F_\beta \), with \( dE_{I-I\beta} \) showing the change in energy, and beyond, in momentum, of the incoming wave, as it goes through the interspersed medium. These pages aim at showing that the macro-scale experimental set-up forecasts results on atomic energy levels, thereby creating a link between the macro and micro atomic scales through an application of relativistic energy equations to the Bohr’s atom model. It is enriched with an approach that uses the Heisenberg’s uncertainty about the momentum of n-type charges having no potential energy in the well but only kinetic energy, all being treated in the angle of the wave-particle duality.

It must be mentioned that the document presents a summary only.

Main equation: 
\[
-2p_n^2c^2 \geq dE_{I-I\beta}^2 - m^2c^4
\]

Let’s have the second term of this inequality as \( X(o) \). In this inequality, variations of \( X(o) \): \( - m^2c^4 \rightarrow 0 \). So we can see that the values of the momentum such as \( p_n \) of the n-phased electrons in the potential well are linked to a value of \( X \), comprised between the limits of the variations of \( X(o) \). Now let’s have \( X \) as a fraction of \( X (o) \) minimum value, as \( X = X (o) \) min. \( n’. \); \( 0 < n’ \leq 1 \)

Definition of the values of \( p_n \): 
\[
-2 p^2 c^2 \geq X (o) n’
\]

In this inequality, the right hand-side body varies between \( - m^2c^4 \rightarrow 0 \).

It comes that \( p_n = \left[ mc/ \sqrt{2} \right] \sqrt{n’} \) with \( v_n = \left[ c/ \sqrt{2} \right] \sqrt{n’} \)
Please note:
- the values of the momenta of the n-phased electrons are of the n’ order;
- the value of the de Broglie wavelength of each of the n-phased electrons is
  \[ \lambda_{db} = \left[ \frac{\hbar \sqrt{2}}{mc} \right] \left[ \frac{1}{\sqrt{n'}} \right] \]

Definition of the value of dp_{I-\beta}: the change in momentum of the incoming wave inferred from dE_{I-\beta}.

In a resolution of n’ order \( dE_{I-\beta}^2 = m^2 c^4 (1-n') = m^2 c^4 n' \), whereby n’ is not defined in a rapport to 1 no more. It results from this that
\[ dp_{I-\beta} = mc \sqrt{n'} \]
\[ d\lambda = \left[ \frac{\hbar}{mc} \right] \left[ \frac{1}{\sqrt{n'}} \right] \]
\[ dF = \left[ \frac{mc^2}{\hbar} \right] \sqrt{n'} \]

Resulting relationship between \( p_n \) and dp_{I-\beta}:
\[ p_n = dp_{I-\beta} \left[ \frac{1}{\sqrt{2}} \right] \]

Moreover, \( \lambda_{db} = \frac{c \sqrt{2}}{dF_{I-\beta}} = \frac{\hbar \sqrt{2}}{dp_{I-\beta}} \); and besides \( \frac{h}{p_n} = \lambda_{db} \), for all n’ values.

Preliminary conclusion:
\[ dE_{I-\beta} \approx E_K \sqrt{2} \quad \text{by} \quad \pm \hbar \Delta dE \]

- The change in momentum of the wave through a medium is closely associating the momentum value and beyond the amplitude of the phased electrons motion in the secondary \( \beta \) wave;
- From the above formula, it comes that, if the wave goes through several media in series, the kinetic energy of the phased n-particles at every stage after new energy absorption is given by:
\[ E_K \approx dE \left[ \sqrt{2} \right]^p \]

whereby \( p \) varies from 1 to infinity, symbolizing each stage of absorption number.

- This equation unifies between the Bohr’s atom model, the Heisenberg’s uncertainty principle and the Schroedinger ‘s postulate of the dot potential energy of a particle in a potential well;
- It also unifies between the micro-atomic and the macro-scale.
- Note that \( p \) corresponds each possible eigen state and shows a clear equivalency to \( n \), which shows the number of energy levels.

Application to an example:
Let’s consider the ionization frequency that must be provided to an electron on the \( n=1 \) level of energy in the Bohr’s atom for it to leave its orbit: it is given by \( dE/h \), whereby \( dE = dE_{I-\beta} \), and the value of the ionization frequency is then : \( dE_{I-\beta} / h \). Well, the curve on the diagram of \( dE_{I-\beta} \) versus \( p \) shows the relationship between the kinetic energy of a particle after the absorption of the amount of energy such as \( dE_{I-\beta} \) in the way that we find the value of its kinetic energy as the next point on the curve crossing with a \( p \) vertical, conformal to the above formula.

If dividing the ordinal values by \( h \), we should obtain of frequency values, and we know that if the particle were held up by some potential barrier, its kinetic energy would be dot in the outwards direction, so that it would still need a new amount of energy to absorb such as \( Fh \), whereby \( F \) is the ionization frequency at the stage \( n \) (initial) + 1. On this diagram, it is visible that \( n = \frac{1}{2} p \), or
alternatively, stands very close, so that following the curve down from left to right leads us to find the value of the kinetic energy or the ionization frequency of a particle at the next stage of absorption such as $p$, with $n = \frac{1}{2} p$, the curve offering a comparison to the Bohr’s model ionisation frequencies.

Please note:

$$E_K \approx dE_{I,\beta} \sqrt{2^n} \approx dE_{I,\beta} \sqrt{2^{2n}} = dE_{I,\beta} \cdot 2^n$$

**Conclusion:**

Within the latitude that is offered by the insertion of the postulate of the kinetic energy of a particle in a potential well, by the insertion of the physical uncertainty of its momentum conformal to the Heisenberg’s demonstration, the Bohr’s atom model avails to stand quite close to the predictions of the links between absorbed energies and subsequent kinetic energies, as could apply to the particles in a potential well, if receiving energy from a resonant isolated system after an incoming wave goes through it, and as could apply through a resolution using relativistic energy equations and in respect for the wave particle duality of the electron.

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